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Research paper

The new solitary solutions of the foam drainage & (2+1) dimensional breaking soliton equations

Emad H. M. Zahran [1] *, Maha S. M. Shehat [2]

[1] *Department of Mathematical and physical Engineering, Benha University, Faculty of Engineering, Shubra, Egypt*
[2] *Department of Mathematics, Zagazig University, Faculty of Science, Zagazig, Egypt*
**E-mail:e_h_zahran@hotmail.com*

Abstract

In this study, the modified extended tanh-function method is handling to obtain many new solitary wave solutions of two important models in nonlinear physics. The first one is the foam drainage equation which is a simple model for describing the flow of liquid through channels and nodes between the bubbles, driven by gravity and capillarity. The second is (2+1)-dimensional breaking soliton equation which describe the interaction of a Riemann wave propagating along the y-axis with along the x-axis. The obtained results are compared with that obtained in previous work.

Keywords: *Foam Drainage Equation; the (2+1)-Dimensional Breaking Soliton Equation; the Modified Extended Tanh-Function Method; Ricatti Equation; Travelling Wave Solution.*

1. Introduction

In fluid mechanics trooping pockets of gas in liquid will form what is mean by the foam whose influence in the properties of liquid is important. The foam drainage in fluid mechanics according to the accessible configuration is modeled analytically by one-dimensional nonlinear partial differential equation. Also the (2+1)-dimensional breaking soliton equation which contributes to understanding most experiments and complex phenomena to many nonlinear branches of physics These two important models which used to represent the wave motion in different branches of nonlinear physics (fluid, plasma,…,etc) are investigated mathematically. The modified extended tanh-function method is a good technique to obtain the exact travelling wave solutions (which contain some parameters) for these equations if these parameters take definite values the solitary wave solution can be derived from it Several methods are applied successively to solve the nonlinear partial equations through many authors [1-22] such as extended Jacobi elliptic function method, the F-expansion method, the modified simple equation-method, the modified simple equation-method, the $(\frac{G'}{G})$ -expansion method ,the modified $(\frac{G'}{G},\frac{1}{G})$ -expansion method , the exp($-\phi(\zeta)$ -expansion method, modified extended exp($-\phi(\zeta)$ −expansion method, the Riccati-Bernoulli Sub-ODE method, exp-function method, the tanh-function method, the extended tanh-function method, modified extended tanh-function method and so on. The main idea of this method is finding the exact solution of any models which can be expressed by a polynomial of $\phi(\zeta)$ which satisfies the Riccati differential equation $\phi' = b+\phi^2(\zeta)$ where $\zeta = x-ct$ while b, c are arbitrary constants to be determined later .The degree of the polynomial can be calculated by the homogenous balance between the highest order derivative term and the nonlinear term. Equating the coefficients of the different power of $\phi(\zeta)$ to zero , we get the system of algebraic equations .The constants of the polynomial can be determined by solving this system of algebraic equation by Maple or any other computer program. The remainder of this paper is organized as follows: In section 2, we provide a description of the modified extended tanh-function method .In section 3 we use this method to find the exact and the solitary wave solutions of the nonlinear evolution equations given above. In section 4 our conclusion is presented.

2. Description of the method

Consider the following nonlinear evolution equation

$$f(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0 \tag{1}$$

Where f is a polynomial in u(x, t) and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method.

Step1. We use the wave transformation,

$$u(x,t) = u(\zeta), \quad \zeta = x-ct \quad and\; c\; is\; cons. \tag{2}$$

To reduce Eq. (1) to the following ODE:

$$p(u, u', u'', u''', ...) = 0 \tag{3}$$

Where P is a polynomial in $u(\xi)$ and its total derivative, while ($'$ $= \frac{d'}{d\zeta}$)

Step2. Suppose the solution of Eq. (3) has the form:

$$u(\xi) = a_0 + \sum_{i=1}^{m}(a_i\phi_i + \frac{b_i}{\phi_i}) \tag{4}$$

Where a_i, b_i are constants to be determined, such that $a_m \neq 0$ or $b_m \neq 0$ and ϕ satisfies the

Riccati equation $\phi' = b + \phi^2$ (5)

Eq. (5) admits several types of solutions according to the value of the constant b namely:
Case1. If $b \prec 0$, then

$$\varphi = -\sqrt{-b}\tanh\left(\sqrt{-b}\zeta\right), or \quad \varphi = -\sqrt{-b}\coth\left(\sqrt{-b}\zeta\right). \tag{6}$$

Case2. If $b \succ 0$, then

$$\varphi = \sqrt{b}\tan\left(\sqrt{b}\zeta\right), or \quad \varphi = -\sqrt{b}\cot\left(\sqrt{b}\zeta\right). \tag{7}$$

Case3. If $b = 0$, then

$$\varphi = -\frac{1}{\zeta} \tag{8}$$

Step3. Determine the positive integer m in Eq. (4) by balancing the highest order derivative term and the nonlinear term Step4. Substitute Eq.(4) along Eq.(5) into Eq.(3) and collecting all the terms of the same power of ϕ^i , $i = 0, \pm1, \pm2, \pm3, ...$ and by equating the coefficient of different power of φ to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i , b_i .
Step5. Substituting these values and the solutions of Eq. (5) into Eq. (4) we obtain the exact solutions of Eq. (1).

3. Application

Example (1): The foam drainage equation [2]
The drainage of foam liquid involves the interplay of gravity, surface tension and viscous force. According to the accessible configuration the dynamic of the foam drainage in fluid mechanics phenomena are modeled analytically using one -dimensional non-linear partial differential equation namely the foam drainage equation.
Here, we will apply the modified extended tanh -function method described in sec.2 to find the exact traveling wave solutions and then the solitary wave solutions for the foam drainage equation.
Consider the foam drainage equation [28]

$$u_t + (u^2 - \frac{\sqrt{u}}{2}u_x)_x = 0 \tag{9}$$

Where x and t scaled position and time coordinates, respectively. Foam is central to a number of everyday activities, both natural and industrial. As such foam has been of great interest for academic research. In the process industries, foam can be a desirable and even essential element of a process. An example is in the case of the forth flotation separation of minerals and coal [28]. Foaming occurs in many distillation and absorption processes. Foams are very important in many technological processes and applications. Their properties are subject to intensive investigational efforts from both practical developers and scientific researchers [28].In this study; we show the effectiveness and convenience of the method by obtaining the exact solution of Eq. (9).
Using the wave variable $\zeta = k(x + ct)$, Eq. (9) is carried to an ODE

$$cku' + k(u^2 - \frac{k}{2}\sqrt{u}u')' = 0 \tag{10}$$

Integrating Eq. (10) with respect to ζ and considering the zero constants for integration we obtain.

$$cku + k(u^2 - \frac{k}{2}\sqrt{u}u') = 0 \tag{11}$$

Then we use the transformation

$$u(\zeta) = v^2(\zeta), \tag{12}$$

That will convert Eq. (11) to

$$kcv^2 + kv^4 - k^2v^2v' = 0 \tag{13}$$

Or equivalently

$$c + v^2 - kv' = 0 \tag{14}$$

Balancing v', v^2 yields m+1 = 2m $\Rightarrow$ m= one. Consequently, we obtain the solution

$$u(\zeta) = a_0 + a_1\varphi(\zeta) + \frac{b_1}{\varphi(\zeta)}, \tag{15}$$

Substituting about v' and v^2 at Eq. (14), and equating different power of ϕ (ζ) to zero, we obtain algebraic system of equation

$$\begin{aligned} a_1^2 - ka_1 &= 0 \\ 2a_0a_1 &= 0 \\ b_1^2 + kbb_1 &= 0 \\ 2a_0b_1 &= 0 \\ c + a_0^2 + 2a_1b_1 - ka_1b_1 - ka_1b + kb_1 &= 0 \end{aligned} \tag{16}$$

Solving this system by Maple or Mathematica, we get

(1) $a_0 = 0,\ a_1 = k \quad ,\ b_1 = -\frac{c}{4k}, b = \frac{c}{4k^2}$

(2) $a_0 = 0,\ a_1 = k,\ b_1 = 0 \quad , b = \frac{c}{a_1^2}$

(3) $a_0 = 0, a_1 = 0\ , b_1 = -\frac{c}{k} \quad , b = \frac{c}{k^2}$

Case (1):

For, $a_0 = 0,\ a_1 = k \quad ,\ b_1 = -\frac{c}{4k}, b = \frac{c}{4k^2}$

The solution is

$$v(\zeta) = k\phi - \frac{c}{4k}(\frac{1}{\phi})$$

When $b<0$, we get the solitary wave solution

$$v(\zeta) = \sqrt{\frac{-c}{4k^2}}\tanh\left(\sqrt{-\frac{c}{4k^2}}\zeta\right) - \frac{c}{4k}\sqrt{-\frac{c}{4k^2}}\cot\left(\sqrt{-\frac{c}{4k^2}}\zeta\right) \tag{17}$$

$$v(\zeta) = \sqrt{\frac{-c}{4k^2}}\coth\left(\sqrt{-\frac{c}{4k^2}}\zeta\right) - \frac{c}{4k}\sqrt{-\frac{c}{4k^2}}\tanh\left(\sqrt{-\frac{c}{4k^2}}\zeta\right) \tag{18}$$

When $b>0$, we get the solitary wave solution

$$v(\zeta)=\sqrt{\frac{c}{4k^{2}}}\tan\left(\sqrt{\frac{c}{4k^{2}}}\,\zeta\right)-\frac{c}{4k}\sqrt{\frac{c}{4k^{2}}}\cot\left(\sqrt{\frac{c}{4k^{2}}}\,\zeta\right) \tag{19}$$

$$v(\zeta)=\sqrt{\frac{c}{4k^{2}}}\cot\left(\sqrt{\frac{c}{4k^{2}}}\,\zeta\right)-\frac{c}{4k}\sqrt{\frac{c}{4k^{2}}}\tan\left(\sqrt{\frac{c}{4k^{2}}}\,\zeta\right) \tag{20}$$

When b=0, we get the trivial solution.
Case (2):

(2) $a_0=0,\ a_1=k,\ b_1=0\ \ ,b=\frac{c}{a_1^{2}}$

The solution is $v(\zeta)=k\,\phi$

When $b<0$, we obtain the solitary wave solution

$$v(\zeta)=-k\sqrt{\frac{-c}{k^{2}}}\tanh\left(\sqrt{-\frac{c}{4k^{2}}}\,\zeta\right) \tag{21}$$

$$v(\zeta)=-k\sqrt{\frac{-c}{k^{2}}}\coth\left(\sqrt{-\frac{c}{k^{2}}}\,\zeta\right) \tag{22}$$

When $b>0$, we obtain the solitary wave solution

$$v(\zeta)=k\sqrt{\frac{c}{k^{2}}}\tan\left(\sqrt{\frac{c}{k^{2}}}\,\zeta\right) \tag{23}$$

$$v(\zeta)=k\sqrt{\frac{c}{k^{2}}}\cot\left(\sqrt{\frac{c}{k^{2}}}\,\zeta\right) \tag{24}$$

When b=0, we get the solitary wave solution.

$$v(\zeta)=-\frac{k}{\zeta} \tag{25}$$

Case (3)

(3) $a_0=0,\ a_1=0,\ b_1=\frac{-c}{k}\ \ ,b=\frac{c}{k^{2}}$

The solution is $v(\zeta)=bk/\phi$
When b<0 , we obtain the solitary wave solution

$$v(\zeta)=\frac{c}{k}\sqrt{\frac{-c}{k^{2}}}\tanh\left(\sqrt{-\frac{c}{k^{2}}}\,\zeta\right) \tag{26}$$

$$v(\zeta)=\frac{c}{k}\sqrt{\frac{-c}{k^{2}}}\coth\left(\sqrt{-\frac{c}{k^{2}}}\,\zeta\right) \tag{27}$$

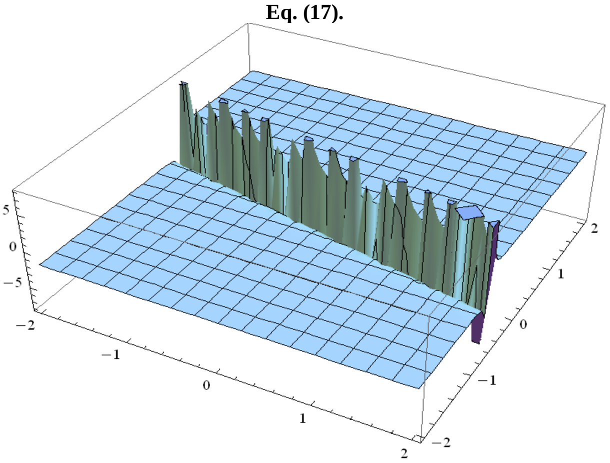
When b>0 , we obtain

$$v(\zeta)=\frac{-c}{k}\sqrt{\frac{c}{k^{2}}}\tan\left(\sqrt{\frac{c}{k^{2}}}\,\zeta\right) \tag{28}$$

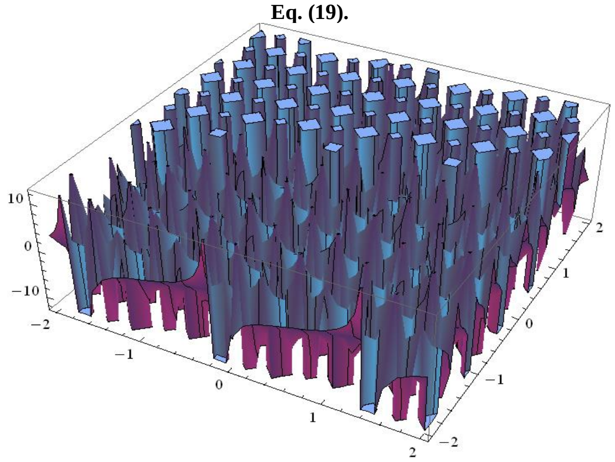
$$v(\zeta)=\frac{-c}{k}\sqrt{\frac{c}{k^{2}}}\cot\left(\sqrt{\frac{c}{k^{2}}}\,\zeta\right) \tag{29}$$

When b=0, we obtain the trivial solution

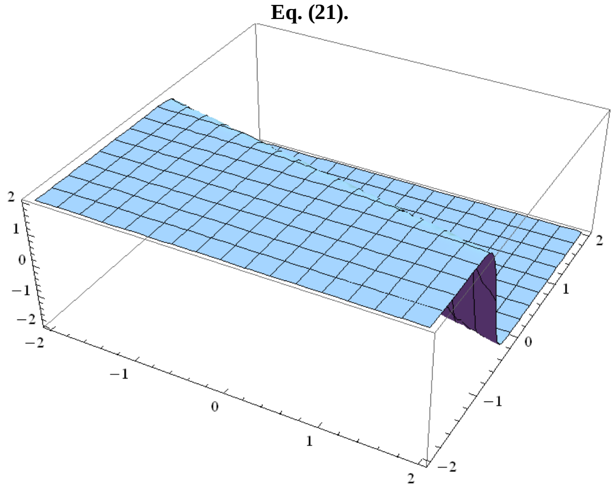
Eq. (17).



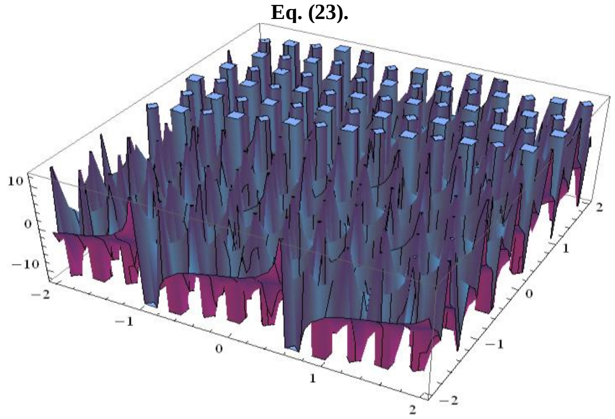
Eq. (19).



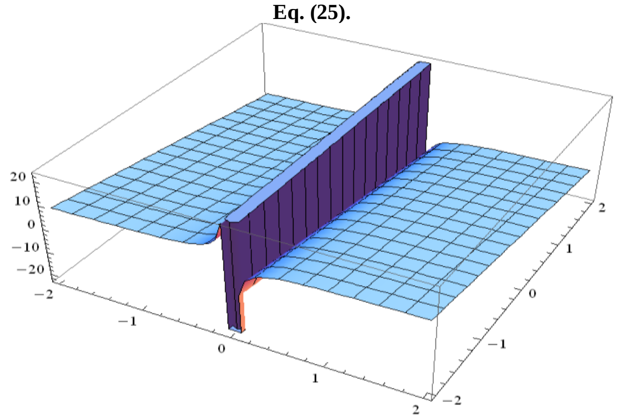
Eq. (21).



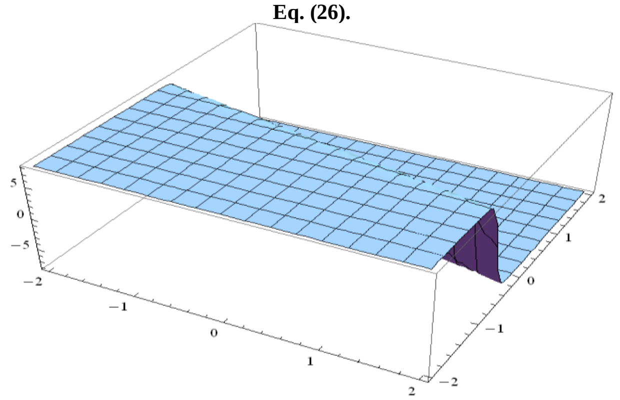
Eq. (23).



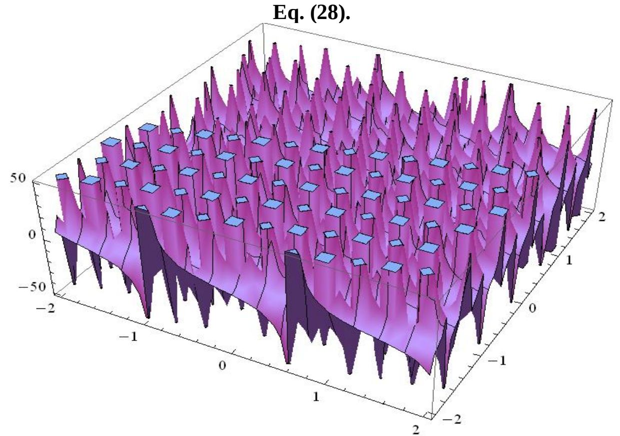
Eq. (25).



Eq. (26).



Eq. (28).



Example (2): The (2+1)-Dimensional Breaking Soliton Equation [29].

Let us consider the (2+1)-dimensional breaking soliton equation,

$$\begin{aligned} &u_t + \alpha u_{xxy} + 4\alpha u v_x + 4\alpha u_x v = 0 \\ &u_y = v_x \end{aligned} \tag{30}$$

Where α is a well-known constant, physically this equation describes the (2+1)-dimensional interaction of a Riemann wave propagating along the y-axis with along wave along the x-axis.
Many authors [30 - 33] study this equation by different manner to get soliton-like solutions, a class of periodic wave solutions, and two classes of exact solutions.
According to our proposed method, let us introduce the transformation $\zeta = x + y - ct$.
Sub.at (30) we get,

$$\begin{aligned} &-cu' + \alpha u''' + 4\alpha u v' + 4\alpha u' v = 0 \\ &u' = v' \end{aligned} \tag{31}$$

Integrating the second part of Eq. (31) with taking the constant of integration in account,

Sub.at the first part of Eq. (31), we obtain

$$-cu' + \alpha u''' + 4\alpha u u' + 4\alpha u'(u + c_1) = 0 \tag{32}$$

Integrating the last Eq., we get

$$(4\alpha c_1 - c)u + \alpha u'' + 4\alpha u^2 = 0 \tag{33}$$

Balancing the nonlinear term with the highest order derivative term, we find m+1 = 2m $\Rightarrow$ m= 2.
Consequently according to the constructed method the solution is

$$u(\zeta) = a_0 + a_1\varphi(\zeta) + \frac{b_1}{\varphi(\zeta)} + a_2\varphi^2(\zeta) + \frac{b_2}{\varphi^2(\zeta)}, \tag{34}$$

Substituting about u, u' *and* u'' at Eq. (33) and equating the coefficients of the different powers of φ to zero, we get the system of algebraic equations namely,

$$\begin{aligned} &6\alpha a_2 + 4\alpha a_2^2 = 0 \\ &2\alpha a_1 + 8\alpha a_1 a_2 = 0 \\ &(4\alpha c_1 - c)a_2 + 8\alpha a_2 b + 4\alpha(a_1^2 + 2a_0 a_2) = 0 \\ &(4\alpha c_1 - c)a_1 + 2\alpha a_1 b + 4\alpha(2a_2 b_1 + 2a_0 a_1) = 0 \\ &6\alpha b_2 b^2 + 4\alpha b_2^2 = 0 \\ &2\alpha b_1 b^2 + 8\alpha b_1 b_2 = 0 \\ &(4\alpha c_1 - c)b_2 + 8\alpha b_2 b + 4\alpha(b_1^2 + 2a_0 b_2) = 0 \\ &(4\alpha c_1 - c)b_1 + 2\alpha b_1 b + 4\alpha(2a_1 b_2 + 2a_0 b_1) = 0 \\ &(4\alpha c_1 - c)a_0 + 4\alpha(a_0^2 + 2a_1 b_1 + 2b_2 a_2) + \alpha(2a_2 b^2 + 2b_1) = 0 \end{aligned} \tag{35}$$

Solving this system of algebraic equations by Maple or any computer program, we get the case

$$a_0 = (\frac{c}{16\alpha} - \frac{c_1}{4}),\ a_1 = 0,\ a_2 - 3/2,\ b = a_0,\ b_1 = 0,\ b_2 = \frac{-3a_0^2}{2}. \tag{36}$$

The solution is

$$u(\zeta) = (\frac{c}{16\alpha} - \frac{c_1}{4}) - (3/2)\varphi^2(\zeta) - \frac{3a_0^2}{2}\frac{1}{\varphi^2(\zeta)}. \tag{37}$$

When b<0

$$\begin{aligned} &u(\zeta) = (\frac{c}{16\alpha} - \frac{c_1}{4}) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})\tanh^2\left(\sqrt{\frac{c_1}{4} - \frac{c}{16\alpha}}\,\zeta\right) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})^2\coth^2\left(\sqrt{\frac{c_1}{4} - \frac{c}{16\alpha}}\,\zeta\right) \\ &u(\zeta) = (\frac{c}{16\alpha} - \frac{c_1}{4}) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})\coth^2\left(\sqrt{\frac{c_1}{4} - \frac{c}{16\alpha}}\,\zeta\right) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})^2\tanh^2\left(\sqrt{\frac{c_1}{4} - \frac{c}{16\alpha}}\,\zeta\right) \end{aligned} \tag{38}$$

When b>0, we get the solitary wave solution

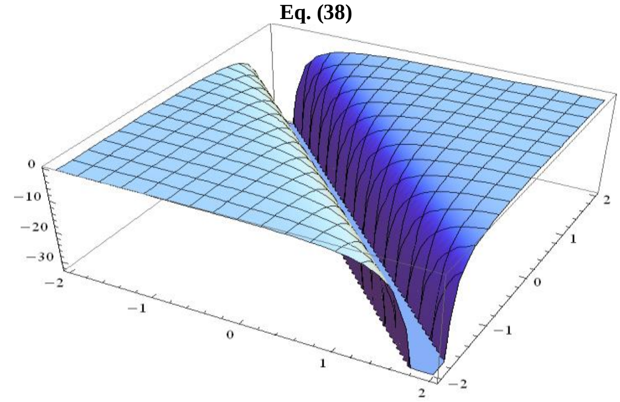
$$u(\zeta) = (\frac{c}{16\alpha} - \frac{c_1}{4}) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})\tan^2\left(\sqrt{\frac{c}{16\alpha} - \frac{c_1}{4}}\,\zeta\right) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})^2\cot^2\left(\sqrt{\frac{c}{16\alpha} - \frac{c_1}{4}}\,\zeta\right) \tag{39}$$

$$u(\zeta) = (\frac{c}{16\alpha} - \frac{c_1}{4}) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})\cot^2\left(\sqrt{\frac{c}{16\alpha} - \frac{c_1}{4}}\,\zeta\right) - \frac{3}{2}(\frac{c}{16\alpha} - \frac{c_1}{4})^2\tan^2\left(\sqrt{\frac{c}{16\alpha} - \frac{c_1}{4}}\,\zeta\right) \tag{40}$$

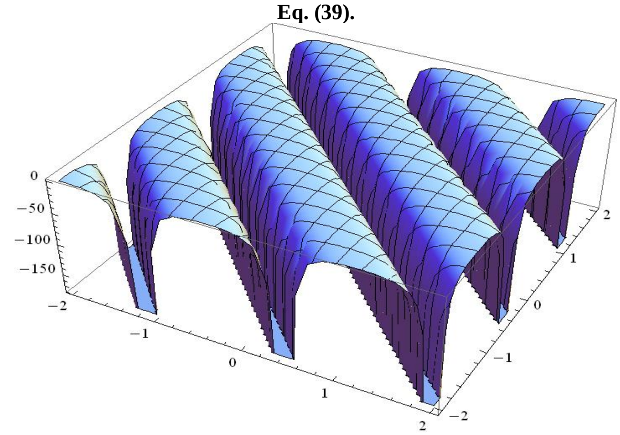
When b=0, we get

$$u(\zeta)=(\frac{c}{16\alpha}-\frac{c_1}{4})-\frac{3}{2\zeta^2}-\frac{3}{2}(\frac{c}{16\alpha}-\frac{c_1}{4})^2\zeta^2 \tag{41}$$

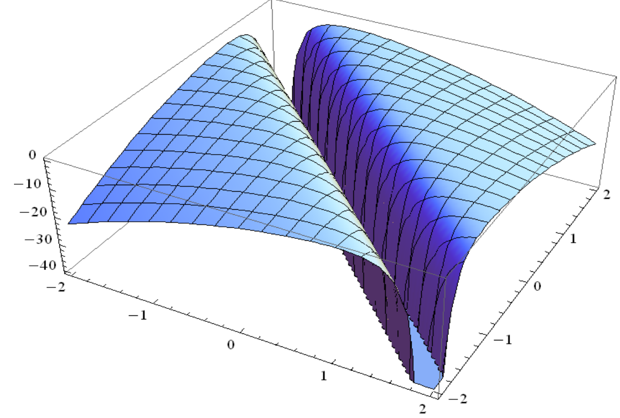
Eq. (38)



Eq. (39).



Eq. (40).



4. Competing interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sector. The author did not have any competing interests in this research.

5. Authors contributions

All parts contained in this research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics, physics and medical science. It represents the research work of the author.

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6. Conclusion

In this study, after comparing our solutions (17-28) for the foam equation and (36) for the (2+1)-dimensional breaking soliton with that obtained by other authors, we find that many new solitary solutions according to the proposed method are introduced which are consistent in some cases but not in other.The proposed method give a large numbers of solitary solutions which give a good interpretation for the foam drainage and the (2+1)-dimensional breaking soliton phenomenon .Also, these large numbers of solitary solutions will help the Scientists to treat these phenomenon experimentally. And this well makes a forward effect on the application in fluid mechanics and plasma physics. Also, the obtained solutions can be considered as benchmarks against the numerical simulations in the fluid mechanics and plasma physics.

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